PROCEEDINGS of IAM, V.8, N.1, 2019, pp.62-75

SUBCLASS OF MEROMORPHICALLY UNIVALENT FUNCTIONS

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Abstract : In this paper, we introduce and study a new subclass $\sigma_A(\alpha,\beta)$ of meromorphically univalent functions with alternating coefficients. We first obtained a necessary and sufficient condition for a function to be in the class $\sigma_A(\alpha,\beta)$. Then we investigate the distortion Theorem, Radius of convexity, convex linear combinations integral transforms and convoltion properties.Furthermore,we studied neighbourhood properties for the class.

Keywords: Meromorphic, Distortion,Radius of convexity and Neighbourhood.

AMS Subject Classification: 30C45.

1. Introduction

Let $\boldsymbol{A}$ denote the class of functions of the form

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disk $E=\{z: z\in E, |z|<1\}$ and satisfy the following usual normalization condition $f(0)=f'(0)-1=0.$ We denote by S the subclass of $\boldsymbol{A}$ consisting of functions $f(z)$ which are all univalent in E.

A function $f\in A$ is a starlike function by the order α, $0\le\alpha<1$ if it satisfy

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha \quad (z\in E). \tag{2}$$

We denote this class with $S^*(\alpha)$.

A function $f \in A$ is a convex function by the order α, $0 \le \alpha < 1$ if it satisfy

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \alpha \qquad (z \in E) \tag{3}$$

We denote this class $K(\alpha)$.

Let T denote the class of functions analytic in E that are of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n, a_n \ge 0 \qquad (z \in E) \tag{4}$$

and let $T^*(\alpha) = T \cap S^*(\alpha)$, $C(\alpha) = \mathrm{T} \cap K(\alpha)$.The class $T^*(\alpha)$ and allied classes possess some interesting properties and have been extensively studied by Silverman [9] and others.

Let σ denote the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \tag{5}$$

which are analytic and univalent in the punctured unit disk $E = \{ z : z \in C \; and \; 0 < |z| < 1 \}$ and which have a simple pole at the orgin with residue 1 there. Let σ_s, $\sigma^*(\alpha)$ and $\sigma_k(\alpha)$ ($0 \le \alpha < 1$) denote the subclasses of σ that are univalent, moromorphically starlike of order α and meromorphically convex of order α respectively. Analytically f(z) of the form (5) is in $\sigma^*(\alpha)$ if and only if

$$\operatorname{Re}\left\{-\frac{zf'(z)}{f(z)}\right\} > \alpha, \; z \in E \tag{6}$$

Similarly, $f \in \sigma_k(\alpha)$ if and only if, f(z) is of the form (5) and satisfies

$$\operatorname{Re}\left\{-\left(1+\frac{zf''(z)}{f'(z)}\right)\right\} > \alpha, \; z \in E \tag{7}$$

It being understood that if α = 1 then $f(z)=\frac{1}{z}$ is the only function which is $\sigma^*(1)$ and $\sigma_k(1)$.

The classes $\sigma^*(\alpha)$ and $\sigma_k(\alpha)$ have been extensively studied by Pommerenke [5], Clunie [3], Royster [7] and others.

A function $f(z)\in\sigma$ is said to be in the class $\sigma(\alpha,\beta)$ if it also satisfy the inequality

$$\operatorname{Re}\left\{z\, f(z)-2\,z^2\, f'(z)\right\}>\beta \tag{8}$$

for some $\alpha\,(\alpha>1)$ and $\beta\,(0\le\beta<1)$ and for all $z\in E$.

Let σ_A be the subclass of σ which consisting of the form

$$f(z)=\frac{1}{z}+a_1z-a_2z^2+a_3z^3-\dots$$

$$=\frac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}a_nz^n,\quad (a_n\ge 0) \tag{9}$$

And let $\sigma_A(\alpha,\beta)=\sigma(\alpha,\beta)\cap\sigma_A$. Motivated by techniques and used similar to these of Silverman [9],Uralegaddi and Gangi [12], Aouf and Darwin [1],Aouf and Hossen [2] and Soybaş, Joshi and Pawar [10,11].

The main objective of this paper is to obtain various interesting properties of functions belong to the class $\sigma_A(\alpha,\beta)$. And also we study some usual properties of the geometric function theory such as coefficient inequalities, distortion theorem , radius of convexity, convex linear combinations, integral transforms,convolution properties and neighbourhood properties for the class .

2. Coefficient Inequalities

In this section we obtain the coefficient inequalities of the function $f(z)$ for the class $\sigma(\alpha,\beta)$.

Theorem 2.1. Let the function *f(z)* defined by (5).

If $\sum_{n=1}^{\infty}(n\alpha-1)a_n \le 1+\alpha-\beta,\ (\alpha>1, 0\le\beta<1)$ (10)

then $f(z)\in\sigma(\alpha,\beta)$

Proof: Let us suppose that the inequality (10) holds true.

Then we have

$$\left|z\ f(z)-\alpha\ z^2\ f'(z)-1-\alpha\right| = \left|-\sum_{n=1}^{\infty}(n\,\alpha-1)a_n\ z^{n+1}\right|$$

$$\le \sum_{n=1}^{\infty}(n\,\alpha-1)a_n\,|z|^{\ n+1}$$

$$\le 1+\alpha-\beta,\ \ (\alpha>1, 0\le\beta<1),$$

which implies that $f(z)\in\sum(\alpha,\beta)$. Hence the theorem.

For functions $\sigma_A(\alpha,\beta)$ the converse of the above theorem is also true.

Theorem 2.2. Let the function $f(z)$ be defined by (1.9).Then *f(z)* in $\sigma_A(\alpha,\beta)$ if and only if

$$\sum_{n=1}^{\infty}(n\,\alpha-1)a_n \le 1+\alpha-\beta, (\alpha>1;\, 0\le\beta<1) \tag{11}$$

Proof. In view of Theorem 2.1 it suffices to show the only it part. Suppose that

$$Re\left\{z\ f(z)-\alpha\ z^2 f'(z)\right\}$$

$$= Re\left\{z\left(\frac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}a_n\ z^n\right)-\alpha\ z^2\left(\frac{-1}{z^2}+\sum(-1)^{n-1}n\,a_n\ z^{n-1}\right)\right\}$$

$$= Re\left\{1+\alpha-\sum_{n=1}^{\infty}(-1)^{n-1}\,(n\,\alpha-1)a_n\ z^{n+1}\right\}>\beta$$

If we choose z to be real and let $z \to 1^-$ we get

$$1+\alpha-\sum_{n=1}^{\infty}(n\,\alpha-1)a_n \le \beta$$

which is equivalent to

$$\sum_{n=1}^{\infty}(n\,\alpha-1)a_n \le 1+\alpha-\beta$$

Corollary 2.3. If $f(z)\in\sigma_A(\alpha,\beta)$ then $a_n \le \dfrac{1+\alpha-\beta}{n\,\alpha-1}$

for $n = 1, 2, \ldots$. Equality holds for the functions of the form

$$f_n(z)=\frac{1}{z}+(-1)^{n-1}\frac{1+\alpha-\beta}{n\,\alpha-1}z^n \tag{12}$$

Theorem 2.4. Let the function *f (z)* defined by $f(z)=\dfrac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}a_n z^n$

$(a_n \ge 0)$ and the function $g(z)$ defined by $g(z)=\dfrac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}b_n z^n\ (b_n>0)$

be in the same class $\sigma_A(\alpha,\beta)$. Then the function *h(z)* defined by

$$h(z)=(1-\lambda)f(z)+\lambda\,g(z)=\frac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}c_n z^n \tag{13}$$

where $c_n=(1-\lambda)a_n+\lambda\,b_n \ge 0,\ (0\le\lambda\le 1)$ is also in the class $\sigma_A(\alpha,\beta)$.

Proof: Suppose that each of the functions *f (z)* and *g (z)* are in the class $\sigma_A(\alpha,\beta)$.

Then making use of (11), we see that

$$\sum_{n=1}^{\infty}(n\,\alpha-1)c_n=\sum_{n=1}^{\infty}(n\,\alpha-1)[(1-\lambda)a_n+\lambda\,b_n] \tag{14}$$

$$=(1-\lambda)\sum_{n=1}^{\infty}(n\,\alpha-1)a_n+\lambda\sum_{n=1}^{\infty}(n\,\alpha-1)b_n$$

$$\leq (1-\lambda)(1+\alpha - \beta) + \lambda(1+\alpha-\beta)$$

$$= 1+\alpha-\beta, \quad (\alpha > 1; 0 \leq \beta < 1, \ 0 \leq \lambda \leq 1)$$

which completes the proof of Theorem .

3. Distortion Theorem

In this section, we prove Distortion Theorem for the class $\sigma_A(\alpha,\beta)$.

Theorem 3.1. If $f(z) \in \sigma_A(\alpha,\beta)$ then for $0 < |z| = r < 1$,

$$\frac{1}{r} - \frac{1+\alpha-\beta}{\alpha-1} r \leq |f(z)| \leq \frac{1}{r} + \frac{1+\alpha-\beta}{\alpha-1} r \tag{15}$$

with equality holds for the function

$$f(z) = \frac{1}{z} + \frac{1+\alpha-\beta}{\alpha-1} z, \quad \text{at } z = ir, r. \tag{16}$$

Proof: Suppose that $f(z) \in \sigma_A(\alpha,\beta)$. In view of the Thorem 2.2

we have
$$\sum_{n=1}^{\infty} a_n \leq \frac{1+\alpha-\beta}{\alpha-1} \qquad (\alpha > 1, 0 \leq \beta < 1). \tag{17}$$

Then for $0 < |z| = r < 1$

$$|f(z)| = \left| \frac{1}{z} + \sum_{n=1}^{\infty} (-1)^{n-1} a_n z^n \right|$$

$$\leq \left|\frac{1}{z}\right| + \sum_{n=1}^{\infty} a_n |z|^n$$

$$\leq \frac{1}{r} + r\sum_{n=1}^{\infty} a_n$$

$$\leq \frac{1}{r} + \frac{1+\alpha-\beta}{\alpha-1} r, \quad \text{by (17)}$$

This gives the right hand side of (15). Also

$$|\ f(z)| = \left|\frac{1}{z} + \sum_{n=1}^{\infty}(-1)^{n-1} a_n\ z^n\right| \geq \frac{1}{r} - \frac{1+\alpha-\beta}{\alpha-1}\, r$$

which gives the left hand side of (15). Hence the theorem.

4. Radius of Convexity

Theorem 4.1. If $f(z) \in \sigma_A(\alpha, \beta)$ then *f(z)* is meromorphically convex of order $\delta,\ (0 \leq \delta < 1)$ in $0 < |z| < r\ (\alpha, \beta, \delta)$, where

$$r(\alpha, \beta, \delta) = \frac{in\ f}{n \geq 1}\left\{\frac{(1-\delta)(n\,\alpha-1)}{\mathrm{n}(1+\alpha - \beta)(n+2-\delta)}\right\}^{1/n+1}, n = 1,2,3....$$

The result is sharp for the function *f (z)* given by

$$f_n(z) = \frac{1}{z} + (-1)^{n-1}\left[\frac{1+\alpha-\beta}{n\,\alpha-1}\right] z^n$$

Proof: Let $f\ (z) \in \sigma_A(\alpha, \beta)$. Then by Theorem 2.2, we have

$$\sum_{n=1}^{\infty} \frac{n\,\alpha-1}{1+\alpha-\beta}\, a_n \leq 1 \tag{18}$$

It is sufficient to show that

$$\left|2 + \frac{z\ f''(z)}{f'(z)}\right| \leq 1 - \delta,$$

or equivalently to show that

$$\left|\frac{f'(z) + (z\ f'(z))'}{f'\,(z)}\right| \leq 1 - \delta, |z| < r\ (\alpha, \beta, \delta) \tag{19}$$

Substituting the series expansion for $f'(z)$ and $(z\ f'(z))'$

in the left side of (19), we have

$$\left|\frac{\sum_{n=1}^{\infty}(-1)^{n-1}\, n\,(n+1)\,a_n\, z^{n-1}}{\frac{-1}{z^2}+\sum_{n=1}^{\infty}(-1)^{n-1} n\, a_n\, z^{n-1}}\right| \leq \frac{\sum_{n=1}^{\infty} n\,(n+1)\,a_n\,|z|^{n+1}}{1-\sum_{n=1}^{\infty} n\,a_n\,|z|^{n+1}}$$

This will be bounded by $1-\delta$ if

$$\sum_{n=1}^{\infty}\frac{n\,(n+2-\delta)}{1-\delta}\,a_n\,|z|^{n+1} \leq 1 \tag{20}$$

In view of (18), it follows that (20) is true if

$$\frac{n\,(n+2-\delta)}{1-\delta}|z|^{n+1} \leq \frac{n\,\alpha-1}{1+\alpha-\beta},\ n=1,2,3.....$$

$$|z| \leq \left\{\frac{(1-\delta)(n\,\alpha-1)}{n\,(n+2-\delta)(1+\alpha-\beta)}\right\}^{1/n+1},\ n=1,2,3..... \tag{21}$$

Setting $|z| = r\,(\alpha,\beta,\delta)$ in (21) the result follows

5. Convex Linear Combinations

In this section, we shall prove that the class $\sigma_A\,(\alpha,\beta)$ is closed under convex linear combinations.

Theorem 6 : Let $f_0(z)=\dfrac{1}{z}$ and

$$f_n(z)=\frac{1}{z}+(-1)^{n-1}\frac{1+\alpha-\beta}{n\,\alpha-1},\ n=1,2,3......$$

Then $f(z)\in\sigma_A(\alpha,\beta)$ if and only if it can be expressed as

$f(z)=\sum_{n=0}^{\infty}\lambda_n\,f_n(z)$, where $\lambda_n \geq 0$ and $\sum_{n=0}^{\infty}\lambda_n = 1$.

Proof. Let $f(z)=\sum_{n=0}^{\infty}\lambda_n f_n(z)$, where $\lambda_n \geq 0$ and $\sum_{n=0}^{\infty}\lambda_n = 1.$

Then $f(z)=\sum_{n=0}^{\infty}\lambda_n \ f_n(z)=\lambda_0 \ f_0(z)+\sum_{n=1}^{\infty}\lambda_n \ f_n(z)$

$$=\left(1-\sum_{n=1}^{\infty}\lambda_n\right) f_0(z)+\sum_{n=1}^{\infty}\lambda_n \ f_n(z)$$

$$=\left(1-\sum_{n=1}^{\infty}\lambda_n\right)\frac{1}{z}+\sum_{n=1}^{\infty}\left(\frac{1}{z}+(-1)^{n-1}\frac{1+\alpha-\beta}{n\alpha-1}z^n\right)$$

$$=\frac{1}{z}+\sum_{n=1}^{\infty}\lambda_n(-1)^{n-1}\frac{1+\alpha-\beta}{n\alpha-1}z^n$$

Since $\sum_{n=1}^{\infty}\frac{n\alpha-1}{1+\alpha-\beta}\lambda_n \ \frac{1+\alpha-\beta}{n\alpha-1}=\sum_{n=1}^{\infty}\lambda_n = 1-\lambda_0 \leq 1$

By theorem 2.2 , $f(z)\in\sigma_A(\alpha,\beta)$

Conversely suppose $f(z)\in\sigma_A(\alpha,\beta)$. since

$$a_n \leq \frac{1+\alpha-\beta}{n\alpha-1}, n=1,2,3 \ \ldots\ldots$$

Setting

$$\lambda_n = \frac{n\alpha-1}{1+\alpha-\beta}, n=1,2,3 \ \ldots\ldots$$

and $\lambda_0 = 1-\sum_{n=1}^{\infty}\lambda_n$, it follows that $f(z)=\sum_{n=0}^{\infty}\lambda_n \ f_n(z)$

Hence the theorem follows.

6. Integral Transforms

In this section, we consider integral transforms of functions in $\sigma_A(\alpha,\beta)$

Theorem 6.1. If *f (z)* is in $\sigma_A(\alpha,\beta)$, then the integral transforms

$$F_c(z)=c\int_0^1 u^c\ f\,(uz)\,du,\ o<c<\infty \tag{22}$$

is in $\sigma_A(\delta)$ where $\delta=\delta\,(\alpha,\beta,c)=\dfrac{2\,(\alpha-c-1)+c\beta}{2\,(\alpha\,c+\alpha-1)-c\beta}$ (23)

The result is best possible for the function

$$f(z)=\frac{1}{z}+\frac{1+\alpha-\beta}{\alpha-1}z,\quad (\alpha>1,0\le\beta<1).$$

Proof: Suppose that $f(z)=\dfrac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}a_n\,z^n\in\sigma_A(\alpha,\beta)$

We have $F_c(z)=c\int_0^1 u^c f(uz)\,du=\dfrac{1}{z}+\sum_{n=1}^{\infty}(-1)^{n-1}\dfrac{c\,a_n}{n+c+1}z^n$

It is sufficient to show that

$$\sum_{n=1}^{\infty}\frac{n+\delta}{1-\delta}\ \frac{c\,a_n}{n+c+1}\le 1\ . \tag{24}$$

Since $f(z)\in\sigma_A\,(\alpha,\beta)$, we have

$$\sum_{n=1}^{\infty}\frac{n\,\alpha-1}{1+\alpha-\beta}a_n\le 1\ . \tag{25}$$

Thus (24) will be satisfied if

$$\frac{(n+\delta)c}{(1-\delta)(n+c+1)}\le\frac{(n\,\alpha-1)}{1+\alpha-\beta}\ \text{ for each n.}$$

$$\delta\le\frac{(n\alpha-1)(n+c+1)-n\,c\,(1+\alpha-\beta)}{c\,(1+\alpha-\beta)+(n\,\alpha-1)(n+c+1)} \tag{26}$$

Since the right hand side of (26) is an increasing function of n, putting n=1 in (26), we get

$$\delta \le \frac{2(\alpha - c - 1) + c\beta}{2(\alpha c + \alpha - 1) - c\beta}$$

The proof of the Theorem is completed.

7. Convolution Properties

Robertson [6] has shown that if $f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n$ and

$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n$ are in *E*, then so is their convolution

$$f(z) * g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n b_n z^n$$

Now we prove the following results for functions in $\sigma_A(\alpha, \beta)$.

Theorem 7.1. If $f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} (-1)^{n-1} a_n z^n$ and $g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} (-1)^{n-1} b_n z^n$

are in $\sigma_A(\alpha, \beta)$, then

$$(f * g)(z) = \frac{1}{z} + \sum_{n=1}^{\infty} (-1)^{n-1} a_n b_n z^n$$

is in the class in $\sigma_A(\alpha, \beta)$

Proof: Suppose $f(z)$ and $g(z)$ are in $\sigma_A(\alpha, \beta)$

By Theorem 2.2, we have

$$\sum_{n=1}^{\infty} \frac{n\alpha - 1}{1 + \alpha - \beta} a_n \le 1 \text{ and } \sum_{n=1}^{\infty} \frac{n\alpha - 1}{1 + \alpha - \beta} b_n \le 1$$

Since $f(z)$ and $g(z)$ are in $\sigma_A(\alpha, \beta)$, so is $(f * g)(z)$.

Further, $\sum_{n=1}^{\infty} \frac{n\alpha-1}{1+\alpha-\beta} \leq \sum_{n=1}^{\infty} \left[\frac{n\alpha-1}{1+\alpha-\beta}\right]^2 a_n b_n$

$$\leq \left[\sum_{n=1}^{\infty} \frac{n\alpha-1}{1+\alpha-\beta} a_n\right] . \sum_{n=1}^{\infty} \left[\frac{n\alpha-1}{1+\alpha-\beta}\right] \leq 1 .$$

Hence by Theorem 2.2, $(f*g)(z)$ is in the class $\sigma_A(\alpha,\beta)$.

This proves the Theorem.

8. Neighborhoods for the class $\sigma_A^{\gamma}(\alpha,\beta)$. .

In this section, we determine the neighborhoods for the class $\sigma_A^{\gamma}(\alpha,\beta)$ which we define as follows:

Definition8.1. A function $f \in \sigma$ is said to be in the class $\sigma_A^{\gamma}(\alpha,\beta)$

if there exists a function $g \in \sigma_A(\alpha,\beta)$ such that

$$\left| \frac{f\ (z)}{g\ (z)} - 1 \right| < 1-\gamma, (z \in E, 0 \leq \gamma < 1) \tag{26}$$

Following the earlier works on neighborhoods of analytic functions by Goodman [4] and Rescheweyh [8], we define the δ - neighborhood of a function $f \in \sigma$ by

$$N_\delta(f) = \left\{ g \in \sigma : g\ (z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \ and \ \sum_{n=1}^{\infty} n \mid a_n - b_n \mid \leq \delta \right\} \tag{27}$$

Theorem 8.2. If $g \in \sigma_A(\alpha,\beta)$ and

$$\gamma = 1 - \frac{\delta\ (\alpha-1)}{\beta-2} \tag{28}$$

then $N_\delta(g) \subset \sigma_A^{\gamma}(\alpha,\beta)$.

Proof. Let $f \in N_\delta(g)$. Then we find from (27) that

$$\sum_{n=1}^{\infty} n \mid a_n - b_n \mid \leq \delta \tag{29}$$

which implies the coefficient inequality $\sum_{n=1}^{\infty} n \mid a_n - b_n \mid \leq \delta, \quad n \in N$.

Since $g \in \sigma_A(\alpha, \beta)$, we have $\sum_{n=1}^{\infty} b_n \leq \dfrac{1+\alpha-\beta}{\alpha-1}$

So that

$$\left| \frac{f(z)}{g(z)} \text{-}1 \right| < \frac{\sum_{n=1}^{\infty} \mid a_n - b_n \mid}{1 - \sum_{n=1}^{\infty} b_n} \leq \frac{\delta \ (\alpha - 1)}{\beta - 2} \quad = 1 - \gamma,$$

provided γ is given by (28). Hence by definition, $f \in \sigma_A^{\gamma}(\alpha, \beta)$ for γ given by (28) which completes the proof.

Acknowledgement : The authors would like to thank the referee for is valuable suggestions to improve the results.

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